

Calculating Outcomes

Brenda Meery

Jen Kershaw

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AUTHORS

Brenda Meery

Jen Kershaw



Fundamental Counting Principle

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[Figure1]

Terry is building his own mountain bike at the bike shop. He has a number of choices to make. First he must choose the color: red, green, blue or purple. Next, he must choose either the normal or extra cushion seat. Then, he must choose his handlebar style, either straight or curved. Terry takes out a piece of paper and tries drawing a picture of the bike with all of his options. How many options does he have?

In this concept, you will learn to calculate outcomes with and without the Counting Principle.

Probability

Probability is a mathematical way of calculating how likely an **event** is to occur. An **event** is a result of an experiment or activity that might include such things as flipping a coin, rolling a die or picking a card from a deck. The number of favorable outcomes is the number of choices (such as rolling a two on a die) and the probability is found by comparing this to the total number of outcomes (six in the case of a die).

Arithmetic can be used to calculate outcomes.

Let's look at an example.

Ichiro's Car Wash offers three different wash services –basic outer body wash, interior cleaning, and finally, custom hand-detailing. Customers order a wash choice plus wax or no wax. You can use a tree diagram to find that there are 6 different choices, or outcomes, for a car wash.

Look at the situation in terms of outcomes.

For the first choice there are 3 different outcomes.

For the second choice there are 2 different outcomes.

The total number of outcomes is $3 \times 2 = 6$.

What if you changed the options? Would this method still work? Let's add on to the car wash problem and see.

Consider Ichiro's oil change choices. Customers can get standard or synthetic oil, filter or no filter, and pay with a coupon or pay the regular price. For each of the three choices there are 2 different outcomes.

The total number of outcomes is $2 \times 2 \times 2 = 8$.

This method of calculating the number of **total outcomes** can be stated as a general rule called the **Counting Principle**. The Counting Principle states that the number of choices or outcomes for two **independent** events, **A** and **B** taken together, is the product of the total number of outcomes for each event.

Total outcomes for A and B = (number of outcomes for A) $\times$ (number of outcomes for B)

Once again, the Counting Principle requires that you take the number of choices or outcomes for two independent events and multiply them together. The product of these outcomes will give you the total number of outcomes for each event.

You can use the Counting Principle to find probabilities of events. The probability of any event is equal to the ratio of favorable outcomes to the total number of equally likely possible outcomes.

$$P(\text{event}) = \frac{\# \text{ of favorable outcomes}}{\text{total outcomes}}$$

Favorable outcomes are the outcomes you are looking for.

Let's look at an example.

Suppose you wanted to know the probability of rolling two number cubes and coming up with a sum of 5.

First, find the total number of outcomes.

To find the number of total outcomes for the two tosses, you can use the Counting Principle. Since each toss of a number cube has 6 different outcomes (1, 2, 3, 4, 5, and 6):

$$\begin{aligned}\text{Total outcomes} &= 6 \times 6 \\ &= 36\end{aligned}$$

Next, find the number of favorable outcomes. In this case the favorable outcomes are outcomes that have a sum of 5.

To figure out this particular problem you must add another step. Now list those 36 outcomes and mark the outcomes that result in a sum of 5.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

[Figure2]

From the table above, you can see that there are 4 possible ways that you can roll two die and get a sum of 5.

Then, use the **$P(\text{event})$** formula to find the probability of rolling a sum of 5.

$$\begin{aligned}
 P(5) &= \frac{4}{36} \\
 &= \frac{1}{9}
 \end{aligned}$$

The answer is $\frac{1}{9}$.

The probability of rolling two number cubes and getting a sum of 5 is $\frac{1}{9}$.

Examples

Example 1

Earlier, you were given a problem about Terry building a bike. Terry is building a bike and can choose from 4 colors (red, green, blue, and purple), 2 seat choices (normal and extra cushion), and 2 types of handle bars (straight and curved).

First, notice how many of each option Terry has to choose from.

Colors: 4

Seats: 2

Handle Bars: 2

Next, use the Counting Principle to find the number of possible outcomes he has.

$$\begin{aligned}\# \text{ outcomes} &= 4 \times 2 \times 2 \\ &= 16\end{aligned}$$

The answer is 16.

Terry has 16 different options for the bike he is building

Example 2

A spinner has colors red, blue, green, and yellow. What is the probability of spinning the spinner twice and having it land on the same color both times?

First, rather than draw a tree diagram, use the Counting Principle to find the number of total outcomes. Since each spin has 4 outcomes:

$$\begin{aligned}\text{Total outcomes} &= 4 \times 4 \\ &= 16\end{aligned}$$

Next, find the number of ways both spinners can land on the same color both times:

	red	blue	green	yellow
red	red - red	blue - red	green - red	yellow - red
blue	red - blue	blue - blue	green - blue	yellow - blue
green	red - green	blue - green	green - green	yellow - green
yellow	red - yellow	blue - yellow	green - yellow	yellow - yellow

[Figure3]

Then, find the ratio of favorable outcomes to total outcomes.

$$\begin{aligned}P(\text{same color}) &= \frac{4}{16} \\ &= \frac{1}{4}\end{aligned}$$

The answer is $\frac{1}{4}$.

The probability of spinning a spinner with four colors twice and getting the same color both times is $\frac{1}{4}$.

Example 3

Jake has two shirts, four pairs of pants and three sweaters. How many different outfits can he make?

First, notice how many of each type of clothing Jake has.

Shirts: 2

Pants: 4

Sweaters: 3

Next, use the Counting Principle to find the number of possible outfits.

$$\begin{aligned}\# \text{ outfits} &= 2 \times 4 \times 3 \\ &= 24\end{aligned}$$

The answer is 24.

Jake has 24 options of outfits to wear.

Example 4

What if Jake has two shirts, five pairs of pants and four sweaters? How many outfits can he make?

First, notice how many of each type of clothing Jake has.

Shirts: 2

Pants: 5

Sweaters: 4

Next, use the Counting Principle to find the number of possible outfits.

$$\begin{aligned}\# \text{ outfits} &= 2 \times 5 \times 4 \\ &= 40\end{aligned}$$

The answer is 40.

Jake has 40 options of outfits to wear.

Example 5

What if Jake only chooses one sweater? How many outfits can he create with four pairs of pants and two shirts?

First, notice how many of each type of clothing Jake has.

Shirts: 2

Pants: 4

Sweaters: 1

Next, use the Counting Principle to find the number of possible outfits.

$$\begin{aligned}\# \text{ outfits} &= 2 \times 4 \times 1 \\ &= 8\end{aligned}$$

The answer is 8.

Jake has 8 options of outfits to wear.

Review

Use the counting principle to determine the number of outcomes.

1. Nigel dropped 3 open-faced peanut butter sandwiches that were equally likely to land face-up or face-down. How many different outcomes are there?

2. Movie Star Toothpaste comes in 3 different flavors: sparkle, blast, and stripe, 3 different sizes, and 2 different tube styles. How many toothpaste choices are there?
3. Dave made 3 predictions for this Sunday's football games. How many different outcomes of being right or wrong are there?
4. Bridget made 4 predictions for this Sunday's football games. How many different outcomes of being right or wrong are there?
5. How many more outcomes will there be if Bridget adds an extra prediction for problem 4 above?
6. Sandy's Sandals come in 4 different models: sport, super-sport, casual, and chic, 5 different colors, and 9 different sizes. How many choices are there?
7. Mike's Bikes features three different bike styles: mountain, racer, and stunt bike. You can choose from 6 different gear systems, 4 frame alloys, and 5 colors. How many bike choices are there?
8. Tilly spins a spinner that has red, blue, and yellow sections 3 times. How many different outcomes are possible?
9. Tilly spins a spinner that has red, blue, and yellow sections and tosses a number cube. How many different outcomes are possible?
10. Tilly tosses a number cube 2 times. How many different outcomes are possible?
11. Tilly tosses a number cube 3 times. How many different outcomes are possible?

Use the counting principle to determine probabilities for a spinner with four colors: red, yellow, blue and green.

12. Kalyani spins the spinner 2 times. What is the probability that the arrow will land on green both times?
13. Kalyani spins the spinner 2 times. What is the probability that the arrow will land on the same color both times?
14. Kalyani spins the spinner 2 times. What is the probability that the arrow will land on a different color each time?

Review (Answers)

Click [HERE](#) to see the answer key or go to the Table of Contents and click on the Answer Key under the 'Other Versions' option.

Resources

Ex: Counting Principle and Probability
 Three boys and three girls stand in line.

a) In how many ways can they line up? **720 ways**
 $P_6 = 720$

b) In how many ways can they line up if the first person in line is a boy and then the line alternates by gender? **36 ways**
 $B \cdot G \cdot B \cdot G \cdot B \cdot G$
 $3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1$

c) Find the probability that the first person in line is a boy and the line alternates by gender:
 $P(BGBGBC) = \frac{36}{720} = \frac{1}{20} = 0.05 = 5\%$

d) Find the probability the gender alternates in the line.
 $P(BGBGBG \text{ or } GBGBGB) = \frac{36 + 36}{720} = \frac{72}{720} = \frac{1}{10} = 0.1 = 10\%$

$P(E) = \frac{\text{Number of favorable ways}}{\text{Total number of ways}}$

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Vocabulary

Language: English ▾

Term	Definition
Event	An event is a set of one or more possible results of a probability experiment.
Favorable Outcome	A favorable outcome is the outcome that you are looking for in an experiment.
Outcome	An outcome of a probability experiment is one possible end result.
Total Outcomes	In probability, the total outcomes are the total number of possible outcomes for the probability experiment.

1.0 REFERENCES

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