

Measurement of Probability

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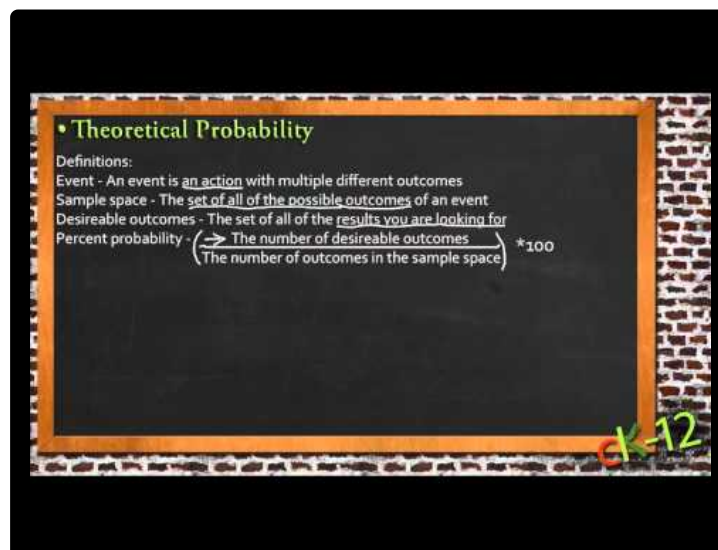
Measurement of Probability

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A **sample space** is the set of all possible outcomes for an event. In tossing a coin, the sample space consists of 2 **outcomes** – getting heads, and getting tails. Each of these outcomes (*heads* and *tails*) could be considered an **event**. Each event has 1 matching element in the **sample space**.

For example, the roll of a single die has 6 possible outcomes: the die can show any number from 1 to 6. We can say that the **sample space** for rolling a single die contains 6 outcomes: {1, 2, 3, 4, 5, 6}. If we say we are interested in rolling a six, then rolling a six is our **event** and this event has 1 matching element in the sample space: {6}. If, on the other hand, we are interested in rolling an even number, then rolling an even number is our event, and this event has 3 matching elements in the sample space: {2, 4, 6}.



<https://www.ck12.org/flx/render/embeddedobject/133128>

Real-World Application: Rolling Dice

A pair of standard, 6-sided dice are rolled, and the total of the numbers that come up determines a player's score. Find the sample space of possible outcomes, and determine how many outcomes result in a score of 5.

The scores a player can get are those in the following set: {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}. But the sample space isn't just this set of 11 events. For example, there's only one way to score 12: the player has to roll a six on each of the dice. But to score 5, the player can roll a 1 and a 4, or a 2 and a 3. Furthermore, there are 2 possibilities for each of these combinations (imagine one die is red, and 1 die is green: we could roll 1 on the red die and 4 on the green or we could roll 4 on the red die and 1 on the green). Even though the dice we actually use may appear identical, they're still separate entities, and it does make a difference which one rolls which number. So there are 4 ways a player can score 5:

(1&4) (2&3) (3&2) (4&1)

To find the full sample space, we must consider all possible outcomes. The best way to do this is with a table (the outcomes that give a score of 5 are highlighted):

		Die number 1 result					
		1	2	3	4	5	6
Die number 2 result	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

The sample space (shown above) has 36 outcomes, 4 of which result in a score of five.

Notice that the number of outcomes in the sample space when you roll 2 dice is the product of the number of outcomes when you roll one die and the number of outcomes when you roll the other die. That is, there are 6 possible outcomes for one die and 6 outcomes for the other die, so there are $6 \times 6 = 36$ possible outcomes when you roll both dice together. This property will be important later.

Find Theoretical Probability of an Event

The theoretical probability of an event is a measure of how likely a given outcome (the *event*) is for a particular experiment, such as tossing a coin. If the experiment were carried out a nearly infinite number of times, the probability of a particular event would be the ratio of how many times a particular outcome occurred to how many times the experiment was performed.

We write the probability that a particular event, E , occurs as:

$$P(E)$$

For example, when tossing a coin we may only be interested in getting heads. We could denote that probability as:

$$P(\textit{Heads}) \quad \text{or simply} \quad P(H)$$

We know that the sample space for tossing a single coin has two elements: *Heads* and *Tails* (or H and T). Each is as likely as the other to occur, so we know that:

$$P(H) = \frac{1}{2}$$

To find the probability of a particular event, we look at how many possible outcomes would contribute to that event, and divide that number by the total number of outcomes in the sample space.

$$P(E) = \frac{\textit{EventSpace}}{\textit{SampleSpace}}$$

When we roll a single 6-sided die, we know that the **chances** of rolling a 3 are 1 in 6. This is, in effect, the probability of rolling 3:

$$P(3) = \frac{1}{6}$$

Calculating Probability

Four coins are tossed simultaneously. What is the probability of getting three or more tails?

We'll start by listing all the possible outcomes in a table. But how many elements will the table have?

Remember that when we rolled 2 dice together, we found the number of outcomes by multiplying the number of outcomes for one die by the number of outcomes for the other die. So if we're flipping four coins, it makes sense to multiply the number of outcomes for each of the four coins together. There are 2 outcomes for each coin, so when all four coins are flipped there should be $2 \times 2 \times 2 \times 2 = 16$ outcomes. Let's organize them as follows:

HHHH	HHHT	HHTH	HHTT
HTHH	HTHT	HTTH	HTTT
THHH	THHT	THTH	THTT
TTHH	TTHT	TTTH	TTTT

Once we fill out the table, we see that there are indeed 16 possible outcomes, and 5 of those outcomes match our **event**. So the probability of getting three or more tails is

$$P(\text{3 or more tails}) = \frac{5}{16}.$$

Find Odds For and Against an Event

When we talk of **probability**, we generally think (as we've seen in this lesson) of the ratio of the number of times our **event** occurs to the number of times the experiment was carried out.

Another way to talk about the chances of an event occurring is with **odds**. You may have heard of the phrases "fifty-fifty" or "even odds" to describe an unpredictable situation like the chances of getting heads when you toss a coin. The phrase means that the coin is as likely to come up tails as it is heads (each event occurring 50% of the time). The **odds** of an event are given by the ratio of the number of times the event occurs to the number of times the event *does not* occur. In *sample space* terms it means:

$$\text{Odds} = \frac{\text{number of matching events in sample place}}{\text{number of non-matching events in sample place}}$$

whereas we would describe probability as

$$\text{Probability} = \frac{\text{number of matching events in sample place}}{\text{number of total events in sample place}}$$

To avoid confusion with probability, odds are usually left as a ratio such as 1:5, which would be read as “one **to** five”. When *probability* is read as a ratio, it’s usually written as a fraction like $\frac{1}{5}$, which would usually be read as “one **in** five.”

Finding the Odds

Find the odds of the following events:

The key to finding odds is looking at how many outcomes result in the event and how many **do not**:

a) Tossing a coin and getting heads.

The sample space consists of 2 outcomes: 1 heads, 1 not-heads. The odds of getting **heads** are 1 : 1 (*one to one*, or **even**).

b) Rolling a die and getting a 3.

The sample space consists of 6 outcomes: 1 three and 5 not-three. The odds of getting **three** are 1 : 5 (*one to five*).

c) Tossing 4 coins and getting exactly 3 tails.

Look back at the example where we found the sample space for tossing 4 coins. The sample space consists of 4 outcomes where exactly 3 tails came up and 12 outcomes when they did not. So the odds of getting **3 tails** are 3 : 12 = 1 : 4 (*one to four*).

Look carefully at part b above. This illustrates the need to avoid confusion between odds and probability. We know that the *probability* of getting a 3 is $P(3) = \frac{1}{6}$ or “one in six,” but the *odds* describes the same event with the ratio 1 : 5 or “one to five”.

Theoretical Probability: An Explanation (Algebra I)



Example

Example 1

Determine the probability of scoring 5 as the combined score of a roll of 2 dice.

We have just seen (in example 2) that there are 4 ways we can score 5 if we roll 2 dice: {(1 & 4), (2 & 3), (3 & 2), (4 & 1)}. The sample space consists of $6 \times 6 = 36$ elements, so the probability of rolling a 5 with 2 dice is $P(5) = \frac{4}{36} = \frac{1}{9}$

Review

For 1-4, find the number of outcomes in the sample space of:

1. Tossing 3 coins simultaneously.
2. Rolling 3 dice and summing the score.
3. Rolling 3 dice and interpreting the result as a 3 digit number.
4. Pulling a card from a standard 52-card deck.

For 5-8, find the theoretical **probability**:

5. Tossing 3 coins simultaneously and getting 2 or more heads.

6. Rolling 3 dice, summing the score and getting 17.
7. Rolling 3 dice and interpreting the result as a 3 digit number, and getting 333.
8. Pulling a club from a standard 52-card deck.

For 9-12, find the **odds**:

9. Tossing 3 coins simultaneously and getting 2 or more heads.
10. Rolling 3 dice, summing the score and getting 12.
11. Rolling 3 dice and interpreting the result as a 3 digit number, and **not** getting 333.
12. Pulling a club from a standard 52-card deck.

Review (Answers)

Click [HERE](#) to see the answer key or go to the Table of Contents and click on the Answer Key under the 'Other Versions' option.

Vocabulary

Language: English ▾

Term	Definition
Probability	Probability is the chance that something will happen. It can be written as a fraction, decimal or percent.
Sample Space	In a probability experiment, the sample space is the set of all the possible outcomes of the experiment.

1.0 REFERENCES

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