

Multinomial Distributions

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Probability distributions for experiments with more than 2 outcomes

Multinomial Distributions

Multinomial Distributions

In later Concepts, we will learn more about multinomial distributions. However, we are now talking about probability distributions, and as such, we should at least see how the problems change for these distributions. We will briefly introduce the concept and its formula here, and then we will get into more detail in later Concepts. Let's start with a problem involving a multinomial distribution.

Multinomial Distributions

- ↳ take an event with multiple possible outcomes (a finite #)
ex. roll a die $\rightarrow 1, 2, 3, 4, 5, 6$
- ↳ repeat event a given number of times

① Roll a die 5 times
Probability of 2 3's and 3 4's

$$\frac{5!}{2! 3!}$$

<https://www.ck12.org/flx/render/embeddedobject/137006>

Calculating Probability

1. You are given a bag of marbles. Inside the bag are 5 red marbles, 4 white marbles, and 3 blue marbles. Calculate the probability that with 6 trials, you choose 3 marbles that are red, 1 marble that is white, and 2 marbles that are blue, replacing each marble after it is chosen.

Notice that this is not a binomial experiment, since there are more than 2 possible outcomes. For binomial experiments, $k = 2$ (2 outcomes). Therefore, we use the binomial experiment formula for problems involving heads or tails, yes or no, or success

or failure. In this problem, there are 3 possible outcomes: red, white, or blue. This type of experiment produces what we call a **multinomial distribution**. In order to solve this problem, we need to use one more formula:

$$P = \frac{n!}{n_1!n_2!n_3!\dots n_k!} \times (p_1^{n_1} \times p_2^{n_2} \times p_3^{n_3} \dots p_k^{n_k})$$

where:

n is the number of trials.

p is the probability for each possible outcome.

k is the number of possible outcomes.

Notice that in this example, k equals 3. If we had only red marbles and white marbles, k would be equal to 2, and we would have a binomial distribution.

The probability of choosing 3 red marbles, 1 white marble, and 2 blue marbles in exactly 6 picks is calculated as follows:

$$n = 6 \text{ (6 picks)}$$

$$p_1 = \frac{5}{12} = 0.416 \text{ (probability of choosing a red marble)}$$

$$p_2 = \frac{4}{12} = 0.333 \text{ (probability of choosing a white marble)}$$

$$p_3 = \frac{3}{12} = 0.25 \text{ (probability of choosing a blue marble)}$$

$$n_1 = 3 \text{ (3 red marbles chosen)}$$

$$n_2 = 1 \text{ (1 white marble chosen)}$$

$$n_3 = 2 \text{ (2 blue marbles chosen)}$$

$$k = 3 \text{ (3 possibilities)}$$

$$P = \frac{n!}{n_1!n_2!n_3!\dots n_k!} \times (p_1^{n_1} \times p_2^{n_2} \times p_3^{n_3} \dots p_k^{n_k})$$

$$P = \frac{6!}{3!1!2!} \times (0.416^3 \times 0.333^1 \times 0.25^2)$$

$$P = 60 \times 0.0720 \times 0.333 \times 0.0625$$

$$P = 0.0899$$

Therefore, the probability of choosing 3 red marbles, 1 white marble, and 2 blue marbles is 8.99%.

2. You are randomly drawing cards from an ordinary deck of cards. Every time you pick one, you place it back in the deck. You do this 5 times. What is the probability of drawing 1 heart, 1 spade, 1 club, and 2 diamonds?

$$n = 5 \text{ (5 trials)}$$

$$p_1 = \frac{13}{52} = 0.25 \text{ (probability of drawing a heart)}$$

$$p_2 = \frac{13}{52} = 0.25 \text{ (probability of drawing a spade)}$$

$$p_3 = \frac{13}{52} = 0.25 \text{ (probability of drawing a club)}$$

$$p_4 = \frac{13}{52} = 0.25 \text{ (probability of drawing a diamond)}$$

$$n_1 = 1 \text{ (1 heart)}$$

$$n_2 = 1 \text{ (1 spade)}$$

$$n_3 = 1 \text{ (1 club)}$$

$$n_4 = 2 \text{ (2 diamonds)}$$

$$k = 4 \text{ (4 possibilities)}$$

$$P = \frac{n!}{n_1!n_2!n_3!\dots n_k!} \times (p_1^{n_1} \times p_2^{n_2} \times p_3^{n_3} \dots p_k^{n_k})$$

$$P = \frac{5!}{1!1!1!2!} \times (0.25^1 \times 0.25^1 \times 0.25^1 \times 0.25^2)$$

$$P = 60 \times 0.25 \times 0.25 \times 0.25 \times 0.0625$$

$$P = 0.0586$$

Therefore, the probability of choosing 1 heart, 1 spade, 1 club, and 2 diamonds is 5.86%.

3. When spinning a spinner, there is an equal chance of landing on orange, green, yellow, red, and black. Suppose you spin the spinner 12 times. What is the probability of landing of orange 2 times, green 3 times, yellow 2 times, red 3 times, and black 2 times?

$$n = 12 \text{ (12 trials)}$$

$$p_1 = \frac{1}{5} = 0.2 \text{ (probability of landing on orange)}$$

$$p_2 = \frac{1}{5} = 0.2 \text{ (probability of landing on green)}$$

$$p_3 = \frac{1}{5} = 0.2 \text{ (probability of landing on yellow)}$$

$$p_4 = \frac{1}{5} = 0.2 \text{ (probability of landing on red)}$$

$$p_5 = \frac{1}{5} = 0.2 \text{ (probability of landing on black)}$$

$$n_1 = 2 \text{ (2 oranges)}$$

$$n_2 = 3 \text{ (3 greens)}$$

$$n_3 = 2 \text{ (2 yellows)}$$

$$n_4 = 3 \text{ (3 reds)}$$

$$n_5 = 2 \text{ (2 blacks)}$$

$$k = 5 \text{ (5 possibilities)}$$

$$P = \frac{n!}{n_1!n_2!n_3!\dots n_k!} \times (p_1^{n_1} \times p_2^{n_2} \times p_3^{n_3} \dots p_k^{n_k})$$

$$P = \frac{12!}{2!3!2!3!2!} \times (0.2^2 \times 0.2^3 \times 0.2^2 \times 0.2^3 \times 0.2^2)$$

$$P = 1,663,200 \times 0.04 \times 0.008 \times 0.04 \times 0.008 \times 0.04$$

$$P = 0.0068$$

Therefore, the probability of landing on orange 2 times, green 3 times, yellow 2 times, red 3 times, and black 2 times is 0.68%.



Example

Example 1

In Austria, 30% of the population has a blood type of O+, 33% has A+, 12% has B+, 6% has AB+, 7% has O-, 8% has A-, 3% has B-, and 1% has AB-. If 15 Austrian citizens are chosen at random, what is the probability that 3 have a blood type of O+, 2 have A+, 3 have B+, 2 have AB+, 1 has O-, 2 have A-, 1 has B-, and 1 has AB-?

$$n = 15 \text{ (15 trials)}$$

$$p_1 = 0.30 \text{ (probability of O+)}$$

$$p_2 = 0.33 \text{ (probability of A+)}$$

$$p_3 = 0.12 \text{ (probability of B+)}$$

$$p_4 = 0.06 \text{ (probability of AB+)}$$

$$p_5 = 0.07 \text{ (probability of O-)}$$

$$p_6 = 0.08 \text{ (probability of A-)}$$

$$p_7 = 0.03 \text{ (probability of B-)}$$

$$p_8 = 0.01 \text{ (probability of AB-)}$$

$$n_1 = 3 \text{ (3 O+)}$$

$$n_2 = 2 \text{ (2 A+)}$$

$$n_3 = 3 \text{ (3 B+)}$$

$$n_4 = 2 \text{ (2 AB+)}$$

$$n_5 = 1 \text{ (1 O-)}$$

$$n_6 = 2 \text{ (2 A-)}$$

$$n_7 = 1 \text{ (1 B-)}$$

$$n_8 = 1 \text{ (1 AB-)}$$

$$k = 8 \text{ (8 possibilities)}$$

$$P = \frac{n!}{n_1!n_2!n_3!\dots n_k!} \times (p_1^{n_1} \times p_2^{n_2} \times p_3^{n_3} \dots p_k^{n_k})$$

$$P = \frac{15!}{3!2!3!2!1!2!1!1!} \times (0.30^3 \times 0.33^2 \times 0.12^3 \times 0.06^2 \times 0.07^1 \times 0.08^2 \times 0.03^1 \times 0.01^1)$$

$$P = 4,540,536,000 \times 0.027 \times 0.1089 \times 0.001728 \times 0.0036 \times 0.07 \times 0.0064 \times 0.03 \times 0.01$$

$$P = 0.000011$$

Therefore, if 15 Austrian citizens are chosen at random, the probability that 3 have a blood type of O+, 2 have A+, 3 have B+, 2 have AB+, 1 has O-, 2 have A-, 1 has B-, and 1 has AB- is 0.0011%.

Review

1. You are randomly drawing cards from an ordinary deck of cards. Every time you pick one, you place it back in the deck. You do this 9 times. What is the probability of drawing 2 hearts, 2 spades, 2 clubs, and 3 diamonds?
2. In question 1, what is the probability of drawing 3 hearts, 1 spade, 1 club, and 4 diamonds?
3. A telephone survey measured the percentage of students in ABC town who watch channels NBX, FIX, MMA, and TSA, respectively. After the survey, analysis showed

that 35% watch channel NBX, 40% watch channel FIX, 10% watch channel MMA, and 15% watch channel TSA. What is the probability that from 8 randomly selected students, 1 will be watching channel NBX, 2 will be watching channel FIX, 3 will be watching channel MMA, and 2 will be watching channel TSA?

4. In question 3, what is the probability that 2 students will be watching channel NBX, 1 will be watching channel FIX, 2 will be watching channel MMA, and 3 will be watching channel TSA?
5. When spinning a spinner, there is an equal chance of landing on pink, blue, purple, white, and brown. Suppose you spin the spinner 10 times. What is the probability of landing of pink 1 time, blue 2 times, purple 2 times, white 2 times, and brown 3 times?
6. In question 5, what is the probability of landing of pink 2 times, blue 2 times, purple 2 times, white 2 times, and brown 2 times?
7. In a recent poll, 23% of the respondents supported candidate A, 19% supported candidate B, 13% supported candidate C, and 45% were undecided. If 7 people are chosen at random, what is the probability that 2 people support candidate A, 2 support candidate B, 1 supports candidate C, and 2 are undecided?
8. In question 7, what is the probability that 2 people support candidate A, 1 supports candidate B, 1 supports candidate C, and 3 are undecided?
9. Suppose you roll a standard die 14 times. What is the probability of rolling a 1 three times, a 2 two times, a 3 one time, a 4 four times, a 5 two times, and a 6 two times?
10. In question 9, what is the probability of rolling a 1 two times, a 2 three times, a 3 three times, a 4 two times, a 5 two times, and a 6 two times?

Review (Answers)

Click [HERE](#) to see the answer key or go to the Table of Contents and click on the Answer Key under the 'Other Versions' option.

Vocabulary

Language: English ▾

Term	Definition
multinomial distribution	A multinomial distribution is a distribution produced where the number of possible outcomes is greater than 2 and where each outcome has a specific probability.

1.0 REFERENCES

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