# Transpose of a Matrix

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# 3.7 Transpose of a Matrix

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Imagine you have a well-organised spreadsheet where every row represents a student and every column represents a test score. Now, suppose you want to see the data from a different perspective - what if you want to compare each test instead of each student? In such a case, you'd flip your data: what was once a row becomes a column and vice versa. This flipping operation is exactly what we call the **transpose** of a matrix.

In mathematics, particularly linear algebra, transposing a matrix is a fundamental operation that reorients the data. It's not only a practical tool in data analysis and computer graphics but also serves as the basis for more advanced concepts such as symmetric matrices and orthogonal projections.

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## Definition of Transpose of Matrix

Given an $m \times n$ matrix $A$ with elements $a_{ij}$ (where $i$ represents the row number and $j$ the column number), the **transpose** of $A,$ denoted by $A^T$ or $A'$, is defined as the $n \times m$ matrix obtained by swapping the rows and columns of $A.$ In other words, the element at position $(i, j)$ in $A$ becomes the element at position $(j, i)$ in $A^T.$

If $A = [a_{ij}],$ then $A^T = [a_{ji}].$

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## Step-by-Step Process for Finding the Transpose

1. **Identify the elements:**
   Look at each element $a_{ij}$ in the original matrix $A.$

2. **Swap the indices:**
   For each element, place $a_{ij}$ in the $j^{th}$ row and $i^{th}$ column of the new matrix $A^T.$

3. **Construct the transposed matrix:**
   Continue this process for all elements to form the new matrix with dimensions $n \times m.$

Let us take a look at an example.

For a matrix: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

- $A$ is a $3 \times 2$ matrix.
- Its transpose $A^T$ will be a $2 \times 3$ matrix:

$$A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

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## Properties of the Transpose

Understanding the properties of the transpose is essential for advanced matrix operations:

### Property 1: Double Transpose Returns the Original Matrix

Taking the transpose of a matrix twice restores the original matrix. This means that for any matrix $A$,

$$\left(A^T\right)^T = A$$

This occurs because the first transpose swaps the rows and columns, and the second transpose swaps them back.

Verification:
Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Compute the transpose of $A$:

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Now, compute the transpose of $A^T$ :

$$\left(A^T\right)^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = A$$

### Property 2: Transpose of a Sum

The transpose of the sum of two matrices equals the sum of their transposes. That is, for any two matrices $A$ and $B$ of the same dimensions,

$$(A+B)^T = A^T + B^T$$

This property holds because transposition affects each element independently, and addition is performed element-wise.

Verification:

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$.

First, add $A$ and $B$:

$$A + B = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

Then, take the transpose:

$$(A+B)^T = \begin{bmatrix} 6 & 10 \\ 8 & 12 \end{bmatrix}$$

Now, compute the transposes individually:

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, \quad B^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}$$

Add these transposes:

$$A^T + B^T = \begin{bmatrix} 1+5 & 3+7 \\ 2+6 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ 8 & 12 \end{bmatrix}$$

Both results are identical.

### Property 3: Transpose of a Scalar Multiple

When a matrix is multiplied by a scalar, taking the transpose is equivalent to multiplying the transpose by the same scalar. For any scalar $k$ and matrix $A$,

$$(kA)^T = kA^T$$

This is because the scalar multiplies with each entry and the order of multiplication is preserved under transposition.

Verification:

Let $k = 3$ and $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

First, multiply $A$ by 3:

$$3A = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$$

Now, take the transpose of $3A$:

$$(3A)^T = \begin{bmatrix} 3 & 9 \\ 6 & 12 \end{bmatrix}$$

Alternatively, compute $A^T$ first:

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

then multiply by 3:

$$3A^T = \begin{bmatrix} 3 & 9 \\ 6 & 12 \end{bmatrix}$$

Both approaches yield the same matrix.

### Property 4: Transpose of a Product

The transpose of a product of two matrices reverses the order of multiplication. That is, for any two matrices $A$ and $B$ (where the product $AB$ is defined),

$$(AB)^T = B^T A^T$$

This reversal occurs because each entry in the product $AB$ involves a sum of products, and transposition interchanges the row and column indices.

Verification:

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Compute the product $AB$ :

$$AB = \begin{bmatrix} (1 \cdot 0 + 2 \cdot 1) & (1 \cdot 1 + 2 \cdot 0) \\ (3 \cdot 0 + 4 \cdot 1) & (3 \cdot 1 + 4 \cdot 0) \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

Then, take the transpose:

$$(AB)^T = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

Now, compute $B^T$ and $A^T$ individually:

$$B^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (\text{Note: } B = B^T), \quad A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}.$$

Then, multiply $B^T$ and $A^T$ :

$$B^T A^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} (0 \cdot 1 + 1 \cdot 2) & (0 \cdot 3 + 1 \cdot 4) \\ (1 \cdot 1 + 0 \cdot 2) & (1 \cdot 3 + 0 \cdot 4) \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

Thus, $(AB)^T = B^T A^T$

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## Case Study: Flipping the Code - Transposing Genes for Deeper Insight

In a biomedical research lab, scientists are collecting data on the expression levels of different genes under multiple experimental conditions. Each row in their dataset corresponds to a gene, and each column corresponds to a specific condition (like temperature or chemical concentration). This gene expression data is initially stored in a matrix $G.$ However, for certain kinds of comparative analysis and visualisation, the scientists need to view the data condition-wise instead of gene-wise. Therefore, they compute the transpose of the gene expression matrix $G^t,$ effectively switching rows and columns so that each row now represents a condition and each column represents a gene. This transposition allows biologists to cluster and analyse conditions more effectively based on how genes respond under them.

Using this scenario and your understanding of the transpose of a matrix, answer the following questions:

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1. If a matrix G is such that G = $G^t$, then G is said to be:

a Symmetric

b Singular

c Skew-symmetric

d Zero Matrix

Check It

## Examples of Transpose of a Matrix

### Example 1

Given the matrix $A = \begin{bmatrix} 2 & 4 & 1 \end{bmatrix}$, find $AA'$, where $A'$ is the transpose of $A$.

Matrix $A$ is a $1 \times 3$ matrix. Its transpose,

$$A' = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix},$$

is a $3 \times 1$ matrix. Therefore, the product $AA'$ will be a $1 \times 1$ matrix (a scalar).

Multiply $A$ by $A'$

$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

Multiply corresponding entries and add:

$$AA' = 2 \times 2 + 4 \times 4 + 1 \times 1 = 4 + 16 + 1 = 21$$

Thus, $AA' = [21]$.

### Example 2

Let $A = \begin{bmatrix} 0 & -3 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$.

Verify the following properties:

(i) $(2A)' = 2A'$

(ii) $(A + B)' = A' + B'$

(iii) $(AB)' = B'A'$

(i) Verification of $(2A)' = 2A'$ :

- Compute $2A$:

$$2A = 2 \times \begin{bmatrix} 0 & -3 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -6 \\ 10 & 2 \end{bmatrix}$$

- Transpose $2A$:

$$(2A)' = \begin{bmatrix} 0 & 10 \\ -6 & 2 \end{bmatrix}$$

- Compute $A'$:

$$A' = \begin{bmatrix} 0 & 5 \\ -3 & 1 \end{bmatrix}$$

- Multiply $A'$ by $2$:

$$2A' = \begin{bmatrix} 0 & 10 \\ -6 & 2 \end{bmatrix}$$

Both computations yield the same result. Thus, $(2A)' = 2A'$.

(ii) Verification of $(A + B)' = A' + B'$ :

- Compute $A + B$:

$$A + B = \begin{bmatrix} 0+2 & -3+4 \\ 5+(-1) & 1+3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 4 \end{bmatrix}$$

- Transpose $A + B$ :

$$(A+B)' = \begin{bmatrix} 2 & 4 \\ 1 & 4 \end{bmatrix}$$

- Compute Transposes Individually:

$$A' = \begin{bmatrix} 0 & 5 \\ -3 & 1 \end{bmatrix}, \quad B' = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$$

- Add $A'$ and $B'$ :

$$A' + B' = \begin{bmatrix} 0+2 & 5+(-1) \\ -3+4 & 1+3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & 4 \end{bmatrix}$$

Both sides are equal. Thus, $(A+B)' = A' + B'$.

(iii) Verification of $(AB)' = B'A'$ :

- Compute $AB$ :

  Multiply $A$ and $B$:

$$AB = \begin{bmatrix} 0 & -3 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$$

  Calculate each entry:

  - Entry $(1,1)$: $0 \times 2 + (-3) \times (-1) = 0 + 3 = 3$
  - Entry $(1,2)$: $0 \times 4 + (-3) \times 3 = 0 - 9 = -9$
  - Entry $(2,1)$: $5 \times 2 + 1 \times (-1) = 10 - 1 = 9$
  - Entry $(2,2)$: $5 \times 4 + 1 \times 3 = 20 + 3 = 23$

  So,

$$AB = \begin{bmatrix} 3 & -9 \\ 9 & 23 \end{bmatrix}$$

- Transpose $AB$ :

$$(AB)' = \begin{bmatrix} 3 & 9 \\ -9 & 23 \end{bmatrix}$$

- Compute $B'$ and $A'$ :

$$B' = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}, \quad A' = \begin{bmatrix} 0 & 5 \\ -3 & 1 \end{bmatrix}$$

- Multiply $B'$ and $A'$ :

$$B'A' = \begin{bmatrix} (2)(0)+(-1)(-3) & (2)(5)+(-1)(1) \\ (4)(0)+(3)(-3) & (4)(5)+(3)(1) \end{bmatrix} = \begin{bmatrix} 0+3 & 10-1 \\ 0-9 & 20+3 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ -9 & 23 \end{bmatrix}$$

Thus, $(AB)' = B'A'$.

## Example 3

Given $A' = \begin{bmatrix} 1 & 2 \\ -2 & 3 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 & 1 \\ -1 & 2 & 3 \end{bmatrix}$, show that,

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

(i) $(A+B)' = A' + B'$

Find $A$ by transposing $A'$ :

$$A = (A')' = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \end{bmatrix}$$

Given $B$ :

$$B = \begin{bmatrix} 4 & 0 & 1 \\ -1 & 2 & 3 \end{bmatrix}$$

Compute $A+B$ :

$$A+B = \begin{bmatrix} 1+4 & -2+0 & 0+1 \\ 2+(-1) & 3+2 & -1+3 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 1 \\ 1 & 5 & 2 \end{bmatrix}$$

Transpose $A+B$ :

$$(A+B)' = \begin{bmatrix} 5 & 1 \\ -2 & 5 \\ 1 & 2 \end{bmatrix}$$

Find $B'$ by transposing $B$ :

$$B' = \begin{bmatrix} 4 & -1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$$

We already have $A'$ given as:

$$A' = \begin{bmatrix} 1 & 2 \\ -2 & 3 \\ 0 & -1 \end{bmatrix}$$

Add $A'$ and $B'$ :

$$A' + B' = \begin{bmatrix} 1+4 & 2+(-1) \\ -2+0 & 3+2 \\ 0+1 & -1+3 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -2 & 5 \\ 1 & 2 \end{bmatrix}$$

Thus, $(A+B)' = A' + B'$.

(ii) $(A-B)' = A' - B'$

Compute $A - B$ :

$$A - B = \begin{bmatrix} 1-4 & -2-0 & 0-1 \\ 2-(-1) & 3-2 & -1-3 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -1 \\ 3 & 1 & -4 \end{bmatrix}$$

Transpose $A - B$ :

$$(A-B)' = \begin{bmatrix} -3 & 3 \\ -2 & 1 \\ -1 & -4 \end{bmatrix}$$

We already have $A'$ and $B'$ given as:

$$A' = \begin{bmatrix} 1 & 2 \\ -2 & 3 \\ 0 & -1 \end{bmatrix} \quad B' = \begin{bmatrix} 4 & -1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$$

Subtract $B'$ from $A'$ :

$$A' - B' = \begin{bmatrix} 1-4 & 2-(-1) \\ -2-0 & 3-2 \\ 0-1 & -1-3 \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ -2 & 1 \\ -1 & -4 \end{bmatrix}$$

Thus, $(A-B)' = A' - B'$.

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**Summary of Transpose of a Matrix**

- The **transpose** of a matrix is obtained by interchanging its rows and columns.
- If matrix $A$ is of size $m \times n$, its transpose $A^T$ will be of size $n \times m$.
- In the transposed matrix, the element at position $(i, j)$ in $A$ becomes the element at $(j, i)$ in $A^T$.
- **Key Properties of Transpose**:
  - Double Transpose: $(A^T)^T = A$
  - Transpose of a Sum: $(A+B)^T = A^T + B^T$
  - Transpose of a Scalar Multiple: $(kA)^T = kA^T$, where $k$ is any scalar
  - Transpose of a Product: $(AB)^T = B^T A^T$

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## Review Questions of Transpose of a Matrix

1. If $A = \begin{bmatrix} 3 & -1 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 4 & -3 \end{bmatrix}$, verify that
   (i) $(A+B)^T = A^T + B^T$,
   (ii) $(A-B)^T = A^T - B^T$,
   (iii) $(2A)^T = 2A^T$.

2. If $A = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ 1 & 0 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$.

3. If $A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 1 \\ 0 & 3 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$.

4. If $A = \begin{bmatrix} 4 & -2 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$.

5. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 2 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$.

6. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, check whether $A^T A = AA^T$.

7. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, verify that $A^T A = I_2$.

8. If $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$, find $A^T - B^T$.

9. If $A = \begin{bmatrix} 5 & 0 & 2 \end{bmatrix}$, find the matrix $AA^T$.

10. If $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, verify that $A^T A = 2I_2$.

# 1.0 REFERENCES

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